NASA TT F-11,499

INVESTIGATION OF A RADIATOR SYSTEM
WITH CONIC HEAT-TRANSFERRING SPIKES

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Translation of "Issledovaniye Izluchatel'noy Sistemy s
Konicheskimi Teplootvodyashchimi Shipami"
Report presented at the Eighteenth Congress of the
International Astronautical Federation, Belgrade,
25-30 September, 1967, 5 pages

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546   JANUARY 1968

SYMBOLS USED

Q	-- heat flux along spike
T_0 and T	-- temperature at base of spike and at distance x from base respectively
α	-- angle at tip of spike
λ	-- heat conductivity coefficient
L	-- length of spike
γ	-- angle between spike axes
σ	-- Stefan-Boltzman radiation constant
ϵ	-- degree of blackness
$E_{ef}(x)$	-- effective radiation density from longitudianl cross-section of spike in question
$E_{ef}(z)$	-- same for neighboring spike (figure 1b)
$E_{inc\ z}(x)$	-- density of incident radiation from neighboring spike at longitudinal cross-section of spike in question
$E^*_{inc\ z}(x)$	-- same, averaged around perimeter of spike in question
$\Sigma E^*_{inc}(x)$	-- incident radiation density from all neighboring spikes, averaged around perimeter of spike in question
$E^*_{ef}(x)$	-- density of effective radiation calculated in consideration of incident radiation averaged around perimeter
r_0	-- radius of body being cooled
δ	-- wall thickness of hollow spike
N	-- heat conductivity parameter of spike $\left(N = \dfrac{2\sigma T_0^3 L}{\lambda} \right)$
n	-- number of spikes in radiator
γ_{min}	-- minimum angle between axes of spikes for given radiation variant
$\overline{\Theta}$	-- effectiveness of radiator system

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ABSTRACT. An analysis is presented of the problem of the
design and optimization of a radiating system consisting
of radially placed conical spikes on an isothermal, cooled
sphere. The case is analyzed when the radius of the sphere
is determined from the condition that the bases of the
most closely placed spikes meet at the surface of the
sphere and the spikes are so placed that the points of
intersection of their axes with the surface of the sphere
are the peaks of a right polyhedron inscribed within the
sphere. Analysis of the relations produced indicates that
the solution of the problem can be reduced to the solution
of a system of two equations with two unknown functions
for the temperature at a given distance from the base of a
spike and the effective radiation density at the given
distance from the base of the spike. The method of calcu-
lation can be extended to include systems with various
groups of spikes under various conditions of radiation
interaction.

In the past, a number of investigations have been published on the $\underline{/1}$[1]
possibility of intensifying heat exchange from heated bodies in a vacuum using
both flat and circular radiating fins. However, in many cases, when the
surface of the body to be cooled may be similar to a polyhedron or a sphere,
the heat radiating elements may be spikes rather than fins.

Let us analyze the problem of the design and optimization of a radiating
system consisting of conical radiating spikes located on an isothermal sphere
to be cooled and extending out into space (figure 1a). We will analyze the
case when: the axes of the spikes pass through the center of the sphere to be
cooled, and the radius of the sphere is determined from the condition that the
bases of the most closely located spikes touch at the surface of the sphere:

$$r_o = \frac{\lambda}{\left[\frac{\sin\left(\frac{\alpha}{2} + \frac{\delta_{min}}{2}\right)}{\sin\frac{\alpha}{2}} - 1 \right]} \tag{1}$$

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[1] Numbers in the margin indicate pagination in the foreign text.

2. The spikes are so located that the points of intersection of their axes with the surface of the sphere are the peaks of a right polyhedron inscribed in the sphere.

We will analyze the problem under the following assumptions:

1. The surface of the body to be cooled not covered with spikes is small in comparison with the surface covered with spikes.

2. The temperature through the cross-section of the spikes is constant, and the surfaces of the spikes are gray diffusion radiators.

3. The heat exchange between any two spikes of the system will be equal to the heat exchange between the longitudinal cross-sections of these spikes with planes perpendicular to the plane in which the axes of the spikes lie /2 (figure 1b). In calculating the mutual irradiation of the spikes, the body being cooled can be replaced by a polyhedron, and the bases of the spikes are assumed to be flat (figure 1c).

4. The surrounding space is a black body with zero temperature.

Considering the above, The law of thermal radiation and the equation for thermal conductivity along the spike will be correct in the following form:

$$Q = -\lambda \frac{dT}{dx} \pi \cdot (L-x)^2 \cdot tg^2 \frac{\alpha}{2} \tag{2}$$

$$dQ = -[E_{ef}^*(x)^2 - \sum E_{inc}^*(x)] \cdot 2\pi (L-x) tg^2 \frac{\alpha}{2} dx \tag{3}$$

Since Q = 0 where x = L, equations (2) and (3) allow us to produce the following expression to determine the distribution of temperature along a spike:

$$\frac{dT}{dx} + \frac{2}{\lambda tg \frac{\alpha}{2}(L-x)^2} \int_x [E_{ef}^*(x) - \sum E_{inc}^*(x)](L-x)dx = 0, \tag{4}$$

$$E_{ef}^*(x) = \varepsilon \sigma T^4 + (1-\varepsilon) \sum E_{inc}^*(x) \tag{5}$$

$$\sum E_{inc}^*(x) = \frac{1}{\pi}[E_{inc z}(x) + E_{inc \gamma}(x) + E_{inc \zeta}(x) + \cdots \tag{6}$$

(the subscripts z, γ, ς, etc. indicate the longitudinal coordinates of the spikes with which the spike being analyzed enters into radiant interactions).

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The boundary condition for (4) is $T = T_0$ where $x = 0$.

Let us find the expression for $\Sigma E^*_{inc}(x)$; to do this, we must first determine the value of $E_{inc\ z}(x)$ for two spikes located at angle γ from each other (figure 1b). We will consider that the density of the incident radiation does not depend on the coordinate y (see figure 1b) and is equal for each value of the average quantity along band $2(L - x)\tan(d/2)$. Then, the expression for the determination of $E_{inc\ z}(x)$ will have the form:

$$E_{inc\ z}(x) = \frac{1}{2\pi}\int_0^{\lambda} \frac{E_{ef}\ (z)(z+\varepsilon_0)(x+\varepsilon_0)\sin^2\gamma \cdot \varphi \cdot dz}{(\lambda-x)[(z+\varepsilon_0)^2+(x+\varepsilon_0)^2-2(x+\varepsilon_0)(z+\varepsilon_0)\cos\gamma]^{3/2}} \qquad (7)$$

where

$$\varphi = [(\lambda-z)+(\lambda-x)]\operatorname{arctg}\frac{[(\lambda-z)tg\frac{\alpha}{2}+(\lambda-x)tg\frac{\alpha}{2}]}{[(z+\varepsilon_0)^2+(x+\varepsilon_0)^2-2(x+\varepsilon_0)(z+\varepsilon_0)\cos\gamma]^{1/2}}$$

$$-[(\lambda-z)-(\lambda-x)]\operatorname{arctg}\frac{[(\lambda-z)tg\frac{\alpha}{2}-(\lambda-x)\cdot tg\frac{\alpha}{2}]}{[(z+\varepsilon_0)^2+(x+\varepsilon_0)^2-2(x+\varepsilon_0)(z+\varepsilon_0)\cos\gamma]^{1/2}} \qquad (8)$$

$$E_{ef}\ (z) = \varepsilon\ G\ T^4+(1-\varepsilon)E_{inc\ x}(z)$$

The incident radiation averaged around the perimeter with cross section x will be determined from the relationship:

$$E^*_{inc\ z}(x) = \frac{1}{\pi}E_{inc\ z}(x) \qquad (9)$$

We note also that for systems with n = 12 and 20, for which the angles between various spikes are not identical, where $\gamma > \gamma_{min}$ the lower limit in expression (7) is a function of x, the form of which can be determined from geometric considerations.

Analysis of the relationships produced shows that the solution of the problem can be reduced to solution of a system of two equations (4) and (5) with two unknown functions $T(x)$ and $E^*_{ef}(x)$. The parameters of the problem for each variant of the spatial placement of the spikes will be: ϵ, α and N. The system of equations (4) and (5) was solved numerically by machine using the method of successive approximations. the results of the solution are shown on figures 2 and 3 in the form of dependences of the effectiveness of the system on ϵ, α and N for various numbers of radiating spikes. The effectiveness of the system means the ratio of the actual radiated flux to the limiting flux which would be radiated by the system with infinitely great heat conductivity

of the spike material and with no radiant interaction between spikes.

The dependences of figures 2 and 3 allow us to determine the flux carried away if we know the geometric dimensions of the system, and also the values of ϵ, T_0 and the heat conductivity of the material. The value $\bar{r}_0$ for each curve on figures 2 and 3 is determined from dependence (1).

The results produced above are also correct for hollow conical spikes (figure 1d) for which there is no heat exchange between the internal surfaces and the thickness of the wall δ is determined from the relationship

$$\frac{(L-x)\,tg\frac{\alpha}{2}-\delta^2}{(L-x)\,tg\frac{\alpha}{2}} = \varphi = const.$$

In this case, when the dependences of figures 2 and 3 are used, parameter N should be replaced by parameter N_1, which is determined from the relationship $N_1 = N/(1 - \phi^2)$.

The dependences of figures 2 and 3 allow us in each concrete case to perform optimization of the radiating system being investigated.

The method of calculation here presented can be extended to systems for which various groups of spikes will find themselves under various conditions, in the process of radiation interaction. In this case, the initial system of equations will include only the integro-differential equations like (4) and integral relationships like (5), as many as there are groups of spikes which differ from each other in their heat balance conditions.
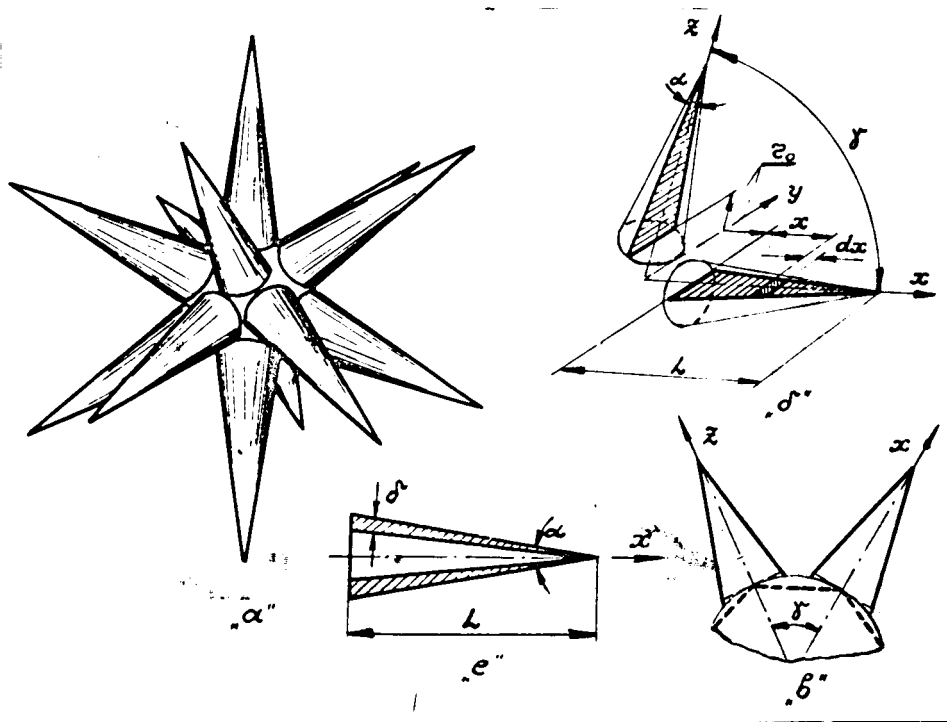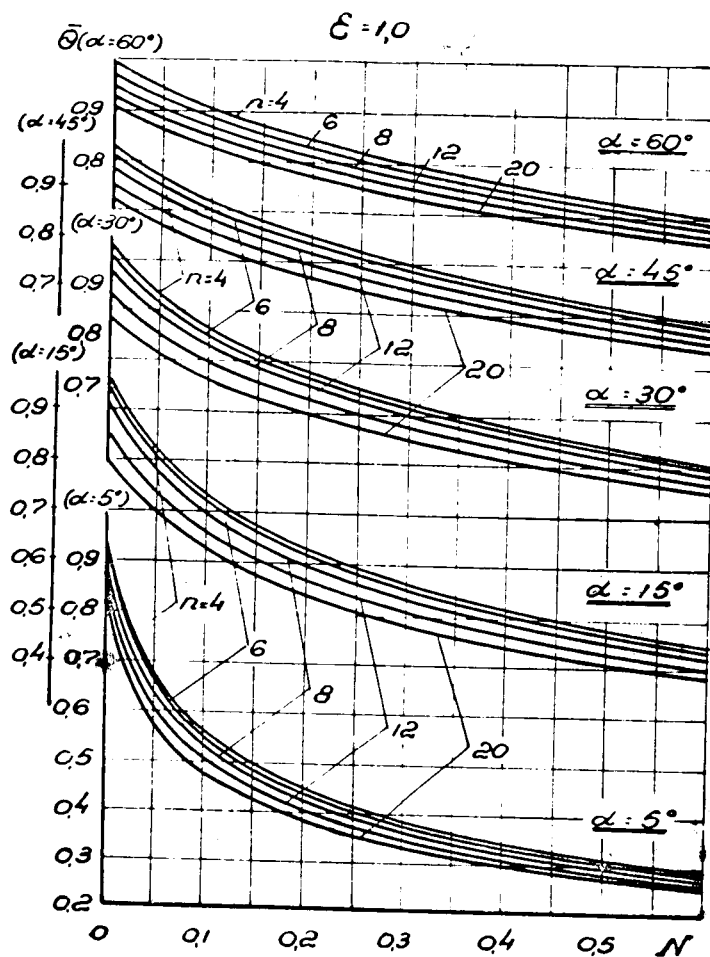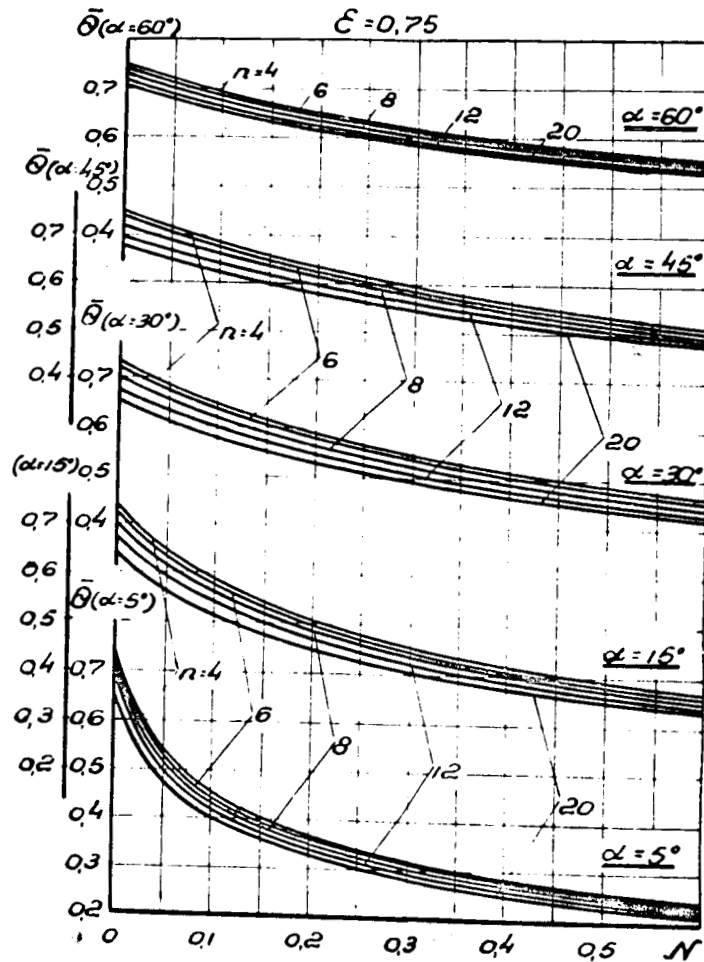
Figure 1

Figure 2

Figure 3